

**A.A.31** Find the intersection of sets (no more than three sets) and/or union of sets (no more than three sets).

**Check Skills You'll Need**

**GO for Help** Lesson NY-3

**Write each set in set-builder notation.**

1.  $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$                       2.  $B = \{1, 3, 5, 7\}$

**Write each set in roster form.**

3.  $C = \{n \mid n \text{ is an even number between } -15 \text{ and } -5\}$   
 4.  $D = \{k \mid k \text{ is a composite number between } 7 \text{ and } 17\}$

**New Vocabulary** • union • intersection • disjoint sets

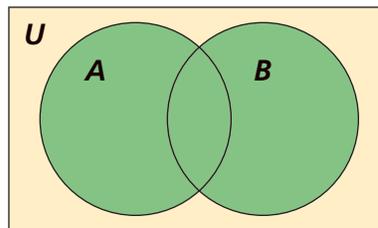
## 1 Operations on Sets

**Vocabulary Tip**

$A \cup B$  is read "A union B," or "the union of sets A and B."

In Lesson NY-3, you examined different ways to express sets. In this lesson, you will perform operations on these sets.

The **union** of two or more sets is the set that contains all elements of the sets. The symbol for union is  $\cup$ . To find the union of two sets, list the elements that are in either set, or in both sets. In the Venn diagram below,  $A \cup B$  is shaded.



**1 EXAMPLE Union of Sets**

A bouquet of flowers contains roses, carnations, and baby's breath. A second bouquet has roses, lilies, and daisies. Both bouquets are put in the same vase. Use union of sets to find the set of flowers in the vase.

- first bouquet:  $B = \{\text{roses, carnations, baby's breath}\}$   
 second bouquet:  $S = \{\text{roses, lilies, daisies}\}$

Write each bouquet as a set.

List the flowers that are in either bouquet, or in both bouquets.

- $B \cup S = \{\text{roses, carnations, baby's breath, lilies, daisies}\}$

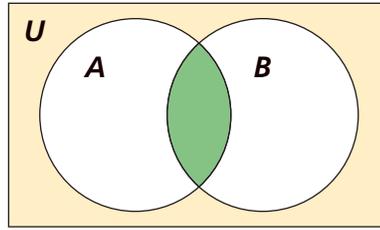
**Quick Check**

- 1  $P = \{5, 10, 15, 20\}$  and  $Q = \{8, 10, 18, 20\}$ . Find  $P \cup Q$ .

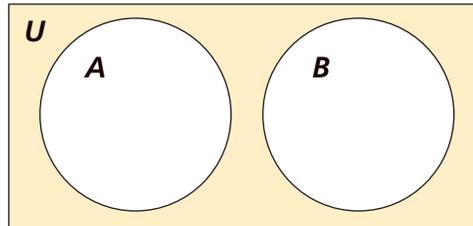
### Vocabulary Tip

$A \cap B$  is read "A intersection B," or "the intersection of sets A and B."

The **intersection** of sets is the set of elements that are common to two or more sets. The symbol for intersection is  $\cap$ . When you find the intersection of two sets, list only the elements that are in both sets. The shaded area below shows  $A \cap B$ .



**Disjoint sets** have no elements in common. The intersection of disjoint sets is the empty set. The diagram below shows two disjoint sets.



### 2 EXAMPLE Intersection of Sets

Set  $C = \{x \mid x \text{ is a natural number less than } 20\}$ , set  $D = \{y \mid y \text{ is an odd integer}\}$ , and set  $E = \{z \mid z \text{ is a multiple of } 4\}$ .

a. Find  $C \cap D$ .

List the elements that are both odd integers and natural numbers less than 20.

$$C \cap D = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$$

b. Find  $C \cap E$ .

List the elements that are both multiples of 4 and natural numbers less than 20.

$$C \cap E = \{4, 8, 12, 16\}$$

c. Find  $D \cap E$ .

$D \cap E = \emptyset$ , or the empty set. There are no multiples of 4 that are also odd, so these are disjoint sets. They have no elements in common.



2  $X = \{2, 4, 6, 8, 10\}$ ,  $Y = \{0, 2, 5, 7, 8\}$ , and  $Z = \{n \mid n \text{ is an odd integer}\}$ .

a. Find  $X \cap Y$ .

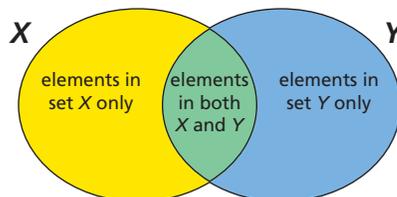
b. Find  $X \cap Z$ .

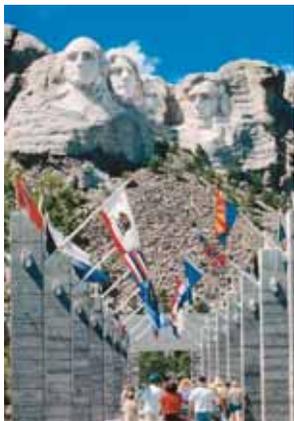
c. Find  $Z \cap Y$ .

### 2

### Solving Problems With Venn Diagrams

You can use Venn diagrams to show the relationships between sets. You can write the elements inside the appropriate section of the diagram. Elements in the intersection of sets appear in the overlapping sections of the Venn diagram.



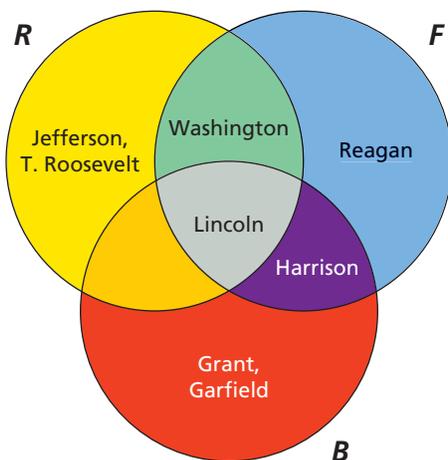


Mt. Rushmore is 1,745 m tall.

### 3 EXAMPLE Venn Diagrams: Three Sets

**Presidents** The set of presidents on Mount Rushmore can be represented by the set  $R = \{\text{Washington, Jefferson, Lincoln, T. Roosevelt}\}$ . The set of presidents born in February can be represented by the set  $F = \{\text{Washington, Harrison, Lincoln, Reagan}\}$ . Presidents who had beards can be represented by the set  $B = \{\text{Lincoln, Grant, Garfield, Harrison}\}$ . Draw a Venn diagram to represent the union and intersection of these sets.

Draw and label three intersecting circles to represent the sets.



- Step 1** Lincoln is in all three sets.  
Put Lincoln in the center area.
- Step 2**  $R \cap F = \{\text{Washington, Lincoln}\}$   
Put Washington in the green area.
- Step 3**  $F \cap B = \{\text{Lincoln, Harrison}\}$   
Put Harrison in the purple area.
- Step 4**  $R \cap B = \{\text{Lincoln}\}$   
The orange area remains empty.
- Step 5** Place the remaining elements of each set into the corresponding yellow, blue, and red sections.

Note that each element only appears in the Venn diagram in one location.



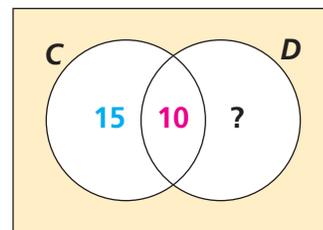
- 3** Let  $X = \{a, e, i, o, u\}$ ,  $Y = \{a, b, c, d, e, f\}$ , and  $Z = \{a, c, e\}$ . Draw a Venn diagram to represent the intersection and union of the sets.

You can also use Venn diagrams to show the number of elements in the union or intersection of a set.

### 4 EXAMPLE Real-World Connection

**Pets** Of 50 cat and dog owners surveyed, 25 have a cat. Ten owners have a dog and a cat. How many owners have a dog?

**Step 1** Draw a Venn diagram. Let  $C =$  owners with cats and  $D =$  owners with dogs.



**Step 2** The intersection of  $C$  and  $D$  represents the owners with both cats and dogs.  
 $C \cap D = 10$ .

**Step 3** Find the number of owners with only a cat.  
 $25 - 10 = 15$ . Enter 15 into the Venn diagram.

**Step 4** The total number of owners is 50. Subtract to find the number of owners with only a dog.  $50 - 15 - 10 = 25$ .

The total number of dog owners is  $25 + 10 = 35$ .

#### Vocabulary Tip

The total number is the union of the two sets,  $C \cup D$ .



- 4** Of 240 college freshmen, 152 are taking history and 81 are taking science and history. How many freshmen are taking history but not science?

# EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

## Practice and Problem Solving

### A Practice by Example



**Example 1**  
(page NY 743)

**Example 2**  
(page NY 744)

**Example 3**  
(page NY 745)



**Example 4**  
(page NY 745)

### B Apply Your Skills

For Exercises 1–12, find each union or intersection. Let  $A = \{2, 5\}$ ,  $B = \{5, 7, 9\}$ ,  $C = \{x \mid x \text{ is an odd number less than } 9\}$ , and  $D = \{x \mid x \text{ is an even number less than } 9\}$ .

- |                |                |                |
|----------------|----------------|----------------|
| 1. $A \cup B$  | 2. $A \cup C$  | 3. $A \cup D$  |
| 4. $B \cup C$  | 5. $B \cup D$  | 6. $C \cup D$  |
| 7. $A \cap B$  | 8. $A \cap C$  | 9. $A \cap D$  |
| 10. $B \cap C$ | 11. $B \cap D$ | 12. $C \cap D$ |

For Exercises 13–16, draw a Venn diagram to represent the union and intersection of the given sets.

13. **Pets** Alex has cats, rabbits, and fish as pets. Becky has cats and dogs. Cory has cats, birds, fish, and turtles. Let  $A = \{\text{cats, rabbits, fish}\}$ ,  $B = \{\text{cats, dogs}\}$ , and  $C = \{\text{cats, birds, fish, turtles}\}$ .
14. Let  $X = \{x \mid x \text{ is a letter in the word ALGEBRA}\}$ ,  $Y = \{y \mid y \text{ is a letter in the word GEOMETRY}\}$ , and  $Z = \{z \mid z \text{ is a letter in the word CALCULUS}\}$ .
15. Let  $P = \{x \mid x \text{ is a prime number less than } 10\}$ ,  $C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ , and  $O = \{a \mid a \text{ is an odd number less than } 10\}$ .
16. Let  $L = \{a, b, c, 1, 2, 3, \text{horse, cow, pig}\}$ ,  $M = \{-1, 0, 1, b, y, \text{pig, duck, } \Delta\}$ , and  $N = \{c, 3, \text{duck, } \Delta\}$ .
17. Of 75 people with cell phones, 42 take pictures with their cell phone and 36 use their cell phone to take pictures and send text messages. How many use their phones to send text messages?
18. Of 100 people in a band, 70 members said they play a sport and 50 members play a sport and take music lessons. How many take music lessons?
19. An ice cream shop owner surveys 200 people who eat chocolate and vanilla ice cream. If 154 people like both flavors, and 196 people like vanilla, how many people like chocolate?

20. **Reasoning** Find two sets  $A$  and  $B$  such that  $A \cup B = \{1, 2, 3, 4, 5\}$  and  $A \cap B = \{2\}$ .
21. **Writing** Let  $M = \{x \mid x \text{ is a multiple of } 3\}$  and  $N = \{x \mid x \text{ is a multiple of } 4\}$ . Describe the intersection of  $M$  and  $N$ .
22. **Critical Thinking** Set  $X$  has 10 elements, set  $Y$  has 15 elements, and  $X \cap Y$  has 5 elements. How many elements are in  $X \cup Y$ ? Explain.

For Exercises 23–25, identify each statement as true or false. Use a Venn diagram or give a counterexample to justify your answer.

23. The intersection of two sets is always a subset of their union.
24. Two sets that contain no elements in common are disjoint sets.
25. The intersection of the set of even numbers and the set of prime numbers is the empty set.

