

Quiz 3 (Answer)

i) $P(n) : 1 + 3 + 5 + \dots + (2n-1) = n^2$

a) $P(1) : 1 + 3 + 5 + \dots + 2(1)-1 = 1^2$

$$P(1) : 2(1)-1 = 1^2$$

$$2-1 = 1^2$$

$$1 = 1$$

The basis step completed and shows the $P(1)$ is true.

b) Inductive hypothesis

$$P(n) : 1 + 3 + 5 + \dots + (2n-1) = n^2$$

$$\rightarrow P(k) : 1 + 3 + 5 + \dots + (2k-1) = k^2$$

c) $P(k+1) : 1 + 3 + 5 + \dots + (2k-1) = k^2$

$$\begin{aligned} P(k+1) &: 1 + 3 + 5 + \dots + (2k-1) + [2(k+1)-1] = k^2 + [2(k+1)-1] \\ &\approx k^2 + 2k + 2 - 1 \\ &\approx k^2 + 2k + 1 \\ &= (k+1)^2 \end{aligned}$$

Both basis step and inductive step completed. So, by the principle of mathematical induction, the statement is true for every positive integer n .

2. $f(0) = 2$

$$f(n+1) = 3f(n) - 2[3-f(n)]$$

$$n = 1, 2, 3$$

a) $f(1)$

$$f(0)+1 = 3f(0) - 2[3-f(0)]$$

$$f(1) = 3(2) - 2(3-2)$$

$$= 6 - 2(1)$$

$$= 4$$

b) $f(2)$

$$f(1)+1 = 3f(1) - 2[3-f(1)]$$

$$f(2) = 3(4) - 2[3-4]$$

$$= 12 + 2$$

$$= 14$$

checked 1/10/18