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DISCRETE MATHEMATICS (DBM2033)
Session December 2017
MODEL EXAM PAPER (QUESTION 4)

Instructions

- Answer ALL questions. Write your answers in the spaces provided.
- Show your working. You may use a non-programmable scientific calculator.

Question 1

Let P(n) be the statement

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

(a) What is the statement P(1)?
(b) Show that P(1) is true, completing the basis step of the proof.
(c) What is the inductive hypothesis?
(d) Complete the inductive step.
(e) Explain why these steps show that this formula is true whatever *n* is a positive integer.
(a) A recurrence relation is given as $a_n = a_{n-2} + a_{n-1}$ where $n \geq 2, a_0 = 7$ and $a_1 = 13$, find a_2, a_3, a_4 and a_5.

Question 2

Prove that

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

for $n \geq 0$.

Question 3

Function f is defined recursively by $f(0) = 1$ and $f(n+1) = 2f(n) - f(n)^2 - 2$ for $n \geq 0$. Find $f(3)+f(4)$.

Question 4

Calculate the first three terms of this recursive function.

$$f(n) = \begin{cases} a_1 = 2 \\ a_n = a_{n-1} + 2 \end{cases}$$