

INSTRUCTION:

Answer all

TOTAL MARKS

NO. TING LIK SIONG

DATE:

Question 1

(a) ~~P(1)~~ $1 = 1^2$
 $1 = 1$

~~to~~Let ~~P(n)~~

P(1) P(2)

(a) $1 + 3 + 5 + \dots + (2n-1) = n^2$

$P(1): 2(1) - 1 = 1^2$

$2 - 1 = 1^2$

$1 = 1$

$P(1)$ is true, ^{because} both side is 1

(b) Inductive hypothesis

$1 + 3 + 5 + \dots + (2n-1) = n^2$

$1 + 3 + 5 + \dots + (2k-1) = k^2$

(c) ~~to~~ Inductive step

$1 + 3 + 5 + \dots + (2k-1) + (k+1) = (k+1)^2$

$(k+1)^2$

$= (k+1)(k+1)$

$(k+1)(k+1)$

$= k^2 + k + k + 1$

$k^2 + k + k + 1$

$= k^2 + 2k + 1$

$k^2 + 2k + 1$

$= k^2 + 2k + 1$

Conclusion:

We have complete the basic step and inductive step, the statement is true for all positive integers n .

Question 2

$$f(0) = 2$$

$$f(n+1) = 3f(n) - 2[3 - f(n)]$$

a) $f(1)$

$$n+1=1$$

$$n=1-1$$

$$=0$$

$$f(0+1) = 3f(0) - 2[3 - f(0)]$$

$$f(1) = 3(2) - 2(3 - 2)$$

$$= 6 - 6 + 4$$

$$f(1) = 4$$

b) $f(2)$

$$n+1=2$$

$$n=2-1$$

$$=1$$

$$f(1+1) = 3f(1) - 2[3 - f(1)]$$

$$= 3(4) - 2(3 - 4)$$

$$= 12 - 6 + 8$$

$$= 14$$

15. X⁶

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Subject: DBM2033 DISCRETE MATH

No:

Date:

1. $P(n) : 1 + 3 + 5 + \dots + (2n-1) = n^2$

a) $P(1) : 1 + 3 + 5 + \dots + 2(1)-1 = 1^2$

$$P(1) : 2(1)-1 = 1^2$$

$$2-1 = 1^2$$

$$1 = 1$$

The basis step completed and ^{shows} the $P(1)$ is true.

b) Inductive hypothesis

$$P(n) : 1 + 3 + 5 + \dots + (2n-1) = n^2$$

$$\rightarrow P(k) : 1 + 3 + 5 + \dots + (2k-1) = k^2$$

c) $P(k) : 1 + 3 + 5 + \dots + (2k-1) = k^2$

$$(2k-1) = k^2 (2k-1)$$

$$(2(1)-1) = k^2 (2(1)-1)$$

$$(2-1) = k^2 (2-1)$$

$$1 = k^2 (1)$$

$$1 = k^2$$

Both basis step and inductive step completed. So, by the principle of mathematical induction, the statement is true for every positive integer n .

2. $f(0) = 2$

$$f(n+1) = 3f(n) - 2[3-f(n)]$$

$$n = 1, 2, 3$$

a) $f(1)$

$$f(0)+1 = 3f(0) - 2[3-f(0)]$$

$$f(1) = 3(2) - 2[3-2]$$

$$= 6 - 2(1)$$

$$= 4$$

b) $f(2)$

$$f(1)+1 = 3f(1) - 2[3-f(1)]$$

$$f(2) = 3(4) - 2[3-4]$$

$$= 12 + 2$$

$$= 14$$

Question 1

a) $P(1) = (2(1) - 1) = 1^2$

Basis step $P(1) = (2(1) - 1) = 1^2$

$2 - 1 = 1^2$

$1 = 1$

In basis step, show that both side is equal to 1, so $P(1)$ is true

b) $1 + 3 + 5 + \dots + (2n - 1) = n^2$

Add k

$1 + 3 + 5 + \dots + (2k - 1) = k^2$

c) $1 + 3 + 5 + \dots + (2k - 1) = k^2$

$P(k) \rightarrow P(k+1)$

$k^2 + (2(k+1) - 1) = (k+1)^2$

$k^2 + 2k + 2 - 1 = (k+1)(k+1)$

$k^2 + 2k + 1 = k^2 + k + k + 1$

$k^2 + 2k + 1 = k^2 + 2k + 1$

$= k^2 + 2k + 1$

Conclusion

We ~~done~~ already done basis step and inductive step. So that is proof for where ~~n integer is true n is all positive~~ n integer integer is true

Question 2

$$f(0) = 2 \text{ and } f(n+1) = 3f(n) - 2[3 - f(n)]$$

~~$$f(1) = f(0+1)$$~~

~~$$f(1) = f(0+1) = 3f(0) - 2[3 - f(0)]$$~~

$$f(1) = f(0+1) = 3(2) - 2[3 - (2)] = 3(2) - 2[1] = 4$$

$$f(2) = f(1+1) = 3(4) - 2[3 - (4)] = 3(4) - 2[-1] = 14$$

$$f(3) = f(2+1) = 3(14) - 2[3 - (14)] = 3(14) - 2[-11] = 64$$

$$f(1) = 4$$

$$f(2) = 14$$

4

Answer.

Question 1

$$\begin{aligned} \text{a) } P(1) \quad (2n-1) &= n^2 \\ 2(1)-1 &= 1^2 \\ 1 &= 1 \end{aligned}$$

Both side is equal therefore $P(1)$ is true.b) # replace n ^{to} with k

$$\begin{aligned} (2n-1) &= n^2 \\ \downarrow \\ (2k-1) &= k^2 \end{aligned}$$

$$\begin{aligned} \text{c) } (2k-1) &= k^2 (2k-1) \\ &= 1 (2k-1) \\ (2k-1) &= (2k-1) \end{aligned}$$

we have completed the both base's and inductive step, by the mathematical principle the $P(1)$ is true.

Question 2

$$\begin{aligned} \text{a) } f(1) &= f(0+1) \\ &= 3(1)(2) - 2[3-1(2)] \\ &= 6-2 \\ &= 4 \end{aligned}$$

$$\begin{aligned} f(1) &= 4 \\ f(2) &= 34 \end{aligned}$$

$$\begin{aligned} \text{b) } f(2) &= f(1+1) \\ &= 3(2)(4) - 2[3-1(4)] \\ &= 24 - (-10) \\ &= 34 \end{aligned}$$

$$P(n) : 1 + 3 + 5 + \dots + (2n-1) = n^2$$

1. a) $P(1) = n^2$ $P(2) = n^2$
 $1 = (1)^2$ $1 + 3 = (2)^2$
 $1 = 1$ $4 = 4$

HW

b) Inductive hypothesis is when all positive value in basis step is true, then the inductive step will be assumed true.

c) Inductive step

assume $n = k$;

$$1 + 3 + 5 + \dots + (2k-1) = k^2$$

$$1 + 3 + 5 + \dots + (2k-1) + (k+1) = k^2 + (k+1)$$

$$k^2 + (k+1) = k^2 + (k+1)$$

$k^2 + (k+1) = k^2 + k + 1$; when both sides are equal, the statement is true.

$$f(n+1) = 3f(n) - 2[3 - f(n)] ; n = 1, 2, 3, \dots ; f(0) = 2$$

2. a) Find $f(1)$

$$\begin{aligned} f(0+1) &= f(1) = 3f(0) - 2[3 - f(0)] \\ &= 3(2) - 2[3 - (2)] \\ &= 6 - 2[1] \\ &= 6 - 2 \\ &= 4 \end{aligned}$$

$$f(0) = 2$$

$$f(-1+1) = 3(-1) - 2[3 - (-1)]$$

$$f(0) = (-3) - 2[3 + 1]$$

$$= (-3) - 2(4)$$

$$= (-3) - 8$$

b) find $f(2)$

$$\begin{aligned} f(1+1) &= f(2) = 3f(1) - 2[3 - f(1)] \\ &= 3(4) - 2[3 - (4)] \\ &= 12 - 2[-1] \\ &= 12 - (-2) \\ &= 12 + 2 \\ &= 14 \end{aligned}$$

Question 1.

$$1+2+3+\dots+k+n(n+1)$$

a) Let $P(n) : 1+3+5+\dots+(2n-1) = n^2$

Basis step : $P(1) = (2 \cdot 1 - 1) = 1^2$
 $= (2(1) - 1) = 1^2$
 $= 1 = 1$
 both side equal to 1

b) Inductive hypothesis

$$P(n) : 1+3+5+\dots+(2n-1) = n^2$$

c) Inductive step: Let $P(n) : 1+3+5+\dots+(2n-1) + [(2n-1)+1] = n^2$
 change n to k

$$P(n) : 1+3+5+\dots+(2k-1) + [(2k-1)+1] = k^2 + [(2k-1)+1]$$

$$(2k-1) + 2k-1+1 = k^2 + 2k - 1 + 1$$

$$2k-1 = k^2 + 2k$$

$$1 = 1$$

After completing the basis step and inductive step, we can conclude that the statement is true where n is all positive integers

Question 2

a) $f(1)$

$$f(0+1) = 3f(0) - 2[3 - f(0)]$$

$$f(1) = 3f(0) - 2[3 - f(0)]$$

$$= 3f(0) - 2[3 - f(0)]$$

$$f(0+1) = 3f(0) - 2[3 - f(0)]$$

$$f(1) = 3f(0) - 2(3-2)$$

$$= 6 - 2(1)$$

$$f(1) = 4$$

b) $f(2)$

$$f(1+1) = 3f(1) - 2[3 - f(1)]$$

$$= 12 - 2(-1)$$

$$= 12 + 2$$

$$f(2) = 14$$

Question 1

(a) $P(1) = (2(1)-1) = (1)^2$
 $= 2-1 = 1$
 $1 = 1$

Since both side has the same value, so $P(1)$ is true.

(b) $1+3+5+\dots+(2k-1)+k+1=(k+1)^2$

(c) $1+3+5+\dots+(2k-1)+k+1=(k+1)^2 + k+1$

$$\begin{aligned}
 &= (k+1)(k+1) + k+1 \\
 &= k^2 + k + k + 1 + k+1 \\
 &= k^2 + 3k + 2 \\
 &= (k+1)(k+2)
 \end{aligned}$$

Since we completed both the basic step and inductive step, so in mathematical conclusion $(k+1)(k+2)$ is true for every positive integer.

Question 2

statement: $F(n+1)$

$$3F(n)-2[3-F(n)]$$

$$F(0)=2 \quad F(1)=4 \quad F(2)=10$$

(a) $F(1) : F(0+1)$

$$= 3F(0)-2[3-F(0)]$$

$$= 3(2)-2[3-(2)]$$

$$= 4(1)$$

$$= 4$$

(b) $F(2) : F(1+1)$

$$= 3F(1)-2[3-F(1)]$$

$$= 3(4)-2[3-(4)]$$

$$= 10$$

$$1 + 3 + 5 + \dots + (2n - 1) = n^2$$

(a) show $p(i)$ is true

basis step

$$p(1): 1 = 1^2$$

$$1 = 1 \quad \text{True}$$

both side must be same

(b) What is the inductive hypothesis?

$$k^2 + 2k + (2 - 1) = (k + 1)(k + 1)$$

(c) complete the inductive step and make conclusion

$$1 + 3 + 5 + \dots + (2k - 1) = k^2$$

$$p(k+1): k^2 + [2(k+1) - 1] = (k+1)^2$$

$$: k^2 + 2k + (2 - 1) = (k + 1)(k + 1)$$

$$: k^2 + 2k + 1 = k^2 + 2k + 1$$

The statement is true whenever the number is positive
~~as long as the number~~

Question 2

$$f(0) = 2$$

a) $f(1) =$

$$f(1) : f(1+1) = 3f(1) - 2[3 - f(1)]$$

$$: f(2) :$$

b) $f(2) =$

$$f(2) : f(2+1) = 3f(2) - 2[3 - f(2)]$$

$$: 3$$

Question 1.

(a) $P(1)$:

$$P1 : 1 + 3 + 5 + \dots + (2n-1) = n^2$$

$$(2(1)-1) = 1^2$$

$$(2-1) = 1$$

$$1 = 1$$

Both sides are equal to 1, so the statement is true.

(b) Inductive hypothesis :

$$1 + 3 + 5 + \dots + (2k-1) = k^2$$

(c.) Inductive step $\Rightarrow$ conclusion :

For each $k \geq 1$ that $P(k)$ implies $P(k+1)$, in other words assuming that inductive hypothesis (b) we can show...

$$1 + 3 + 5 + \dots + (2k-1) + (2(k+1)-1) = (k+1)^2$$

Prove :

$$1 + 3 + 5 + \dots + (2k-1) + (2(k+1)-1) = k^2 + (2(k+1)-1)$$

$$= k^2 + (2k+2-1)$$

$$= k^2 + (2k-1)$$

$$= 2k^2 - k^2$$

$$= k^2$$

Conclusion :

Since we have completed both basis step and inductive hypothesis, by the principle of mathematical induction, the statement for every positive integer n is true.

Question 2.

$$f(0) = 2,$$

$$f(n+1) = 3f(n) - 2[3 - f(n)]$$

a) $f(1)$:

~~$$f(0+1) = 3f(1) - 2[3 - f(1)]$$~~

~~$$f(1) = 3f - 2(3 - f)$$~~

~~$$= 3f - 6 + 2f$$~~

~~$$= 5f - 6$$~~

~~$$f(1) = 3f(1) - 2[3 - f(1)]$$~~

~~$$= 3f - 2(3 - f)$$~~

~~$$= 3f + 6f = 3f + 2(2f)$$~~

~~$$= 9f$$~~

~~$$= 3f + 4f$$~~

~~$$f(1) = 9$$~~

$$f(1) = 3f(1) - 2[3 - f(1)]$$

$$= 3f - 2[3 - f]$$

$$= 3f - 2(3 - f)$$

$$= 3f + 6f$$

$$= 9$$

$$f(1) = 9.$$

b) $f(2)$:

$$f(2) = 3f(2) - 2[3 - f(2)]$$

$$= 6f - 2(3 - 2f)$$

$$= 6f - 2(f)$$

$$= 6f - 2f$$

$$= 4$$

$$f(2) = 4$$